

CHAPTER - VI

PORTFOLIO THEORY

Markowitz portfolio theory

The author of the modern portfolio theory is Harry Markowitz who introduced the analysis of the portfolios of investments in his article “Portfolio Selection” published in the Journal of Finance in 1952. The new approach presented in this article included portfolio formation by considering the expected rate of return and risk of individual stocks and, crucially, their interrelationship as measured by correlation. Prior to this investors would examine investments individually, build up portfolios of attractive stocks, and not consider how they related to each other. Markowitz showed how it might be possible to better of these simplistic portfolios by taking into account the correlation between the returns on these stocks. The diversification plays a very important role in the modern portfolio theory. Markowitz approach is viewed as a single period approach: at the beginning of the period the investor must make a decision in what particular securities to invest and hold these securities until the end of the period. Because a portfolio is a collection of securities, this decision is equivalent to selecting an optimal portfolio from a set of possible portfolios. Essentiality of the *Markowitz portfolio theory is the problem of optimal portfolio selection*. The method that should be used in selecting the most desirable portfolio involves the use of *indifference curves*. Indifference curves represent an investor's preferences for risk and return. These curves should be drawn, putting the investment return on the vertical axis and the risk on the horizontal axis. Following Markowitz approach, the sure for investment return is expected rate of return and a measure of risk is standard deviation (these statistic measures we discussed in previous chapter, section 2.1). The exemplified map of indifference curves for the individual risk-averse investor is presented in Fig.3.1. Each indifference curve here (I) represents the most desirable investment or investment portfolio for an individual investor. That means, that any of investments (or portfolios) plotted on the indifference curves (A, B, C or D) are equally desirable to the investor.

Features of indifference curves:

- ✓ All portfolios that lie on a given indifference curve are equally desirable to the investor.
- An implication of this feature: indifference curves cannot intersect.

- ✓ An investor has an infinite number of indifference curves. Every investor can represent several indifference curves (for different investment tools).
- ✓ Every investor has a map of the indifference curves representing his or her preferences for expected returns and risk (standard deviations) for each

Two important fundamental assumptions than examining indifference curves and applying them to Markowitz portfolio theory:

1. The investors are assumed to prefer higher levels of return to lower levels of return, because the higher levels of return allow the investor to spend more on consumption at the end of the investment period. Thus, given two portfolios with the same standard deviation, the investor will choose the potential portfolio. Portfolio with the higher expected return. This is called an **assumption of non satiation**.

2. Investors are risk averse. It means that the investor when given the choice will choose the investment or investment portfolio with the smaller risk. This is called **assumption of risk aversion**. In reality there are an infinite number of portfolios available for the investment. Is it means that the investor needs to evaluate all these portfolios on return and risk basis? Markowitz portfolio theory answers this question using **efficient set theorem**: *an investor will choose his/her optimal portfolio from the set of the portfolios that (1) offer maximum expected return for varying level of risk, and (2) offer minimum risk for varying levels of expected return.*

Efficient set of portfolios involves the portfolios that the investor will find optimal ones. These portfolios are lying on the “northwest boundary” of the feasible set and are called an **efficient frontier**. The efficient frontier can be described by the curve in the risk-return space with the highest expected rates of return for each level of risk.

Feasible set is opportunity set, from which the efficient set of portfolio can be identified. The feasibility set represents all portfolios that could be formed from the number of securities and lie either on or within the boundary of the feasible set. Considering the assumptions of nonfiction and risk aversion discussed earlier in this section, only those portfolios lying between points A and B on the boundary of feasibility set investor will find the optimal ones. All the other portfolios in the feasible set are inefficient portfolios. Furthermore, if a risk-free investment is introduced into the universe of assets, the efficient frontier becomes the tangential line. This line is called the **Capital Market Line (CML)** and the portfolio at the point at which it is tangential (point M) is called the **Market Portfolio**.

The Expected Rate of Return and Risk of Portfolio

Following Markowitz efficient set portfolios approach an investor should evaluate alternative portfolios inside feasibility set on the basis of their expected returns and standard deviations using indifference curves. Thus, the methods for calculating expected rate of return and standard deviation of the portfolio must be discussed.

The expected rate of return of the portfolio can be calculated in some alternative ways. The Markowitz focus was on the end-of-period wealth (terminal value) and using these expected end-of-period values for each security in the portfolio the expected end-of-period return for the whole portfolio can be calculated. But the portfolio really is the set of the securities thus the expected rate of return of a portfolio should depend on the expected rates of return of each security included in the portfolio. This alternative method for calculating **the expected rate of return on the portfolio** ($E(r)_p$) is the weighted average of the expected returns on its component securities:

$$E(r)_p = \sum_{i=1}^n w_i * E_i(r) = w_1 E_1(r) + w_2 * E_2(r) + \dots + w_n * E_n(r),$$

Here

w_i - the proportion of the portfolio's initial value invested in security i ;

$E_i(r)$ - the expected rate of return of security i ;

n - The number of securities in the portfolio.

Because a portfolio's expected return is a weighted average of the expected returns of its securities, the contribution of each security to the portfolio's expected rate of return depends on its expected return and its proportional share from the initial portfolio's market value (weight). Nothing else is relevant. The conclusion here could be that the investor who simply wants the highest possible expected rate of return must keep only one security in his portfolio which has a highest expected rate of return. But why the majority of investors don't do so and keep several different securities in their portfolios? Because they try to diversify their portfolios aiming to reduce the investment portfolio risk.

Risk of the portfolio As we know from chapter 2, the most often used measure for the risk of investment is standard deviation, which shows the volatility of the securities actual return from their expected return. If a portfolio's expected rate of return is a weighted average of the expected rates of return of its securities, the calculation of standard deviation for the portfolio can't simply use the same approach. The reason is that the relationship between the securities in

the same portfolio must be taken into account. As it was discussed in chapter 2, the relationship between the assets can be estimated using the covariance and coefficient of correlation. As covariance can range from “-” to “+” infinity, it is more useful for identification of the direction of relationship (positive or negative), coefficients of correlation always lies between -1 and +1 and is the convenient measure of intensity and direction of the Relationship between the assets.

Risk of the portfolio, which consists of 2 securities (A and B):

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B K_{AB} \sigma_A \sigma_B \quad (3.2)$$

where:

w_A and w_B - the proportion of the portfolio's initial value invested in security A and B ($w_A + w_B = 1$);

σ_A and σ_B - standard deviation of security A and B;

K_{AB} - coefficient of correlation between the returns of security A and B.

Capital Asset Pricing Model (CAPM)

CAPM was developed by W. F. Sharpe. CAPM simplified Markowitz's Modern **Portfolio** theory, made it more practical. Markowitz showed that for a given level of expected return and for a given feasible set of securities, finding the optimal portfolio with the lowest total risk, measured as variance or standard deviation of portfolio returns, requires knowledge of the covariance or correlation between all possible security combinations. When forming the diversified portfolios consisting large number of securities investors found the calculation of the portfolio **risk** using standard deviation technically complicated.

Measuring Risk in CAPM is based on the identification of two key components of total risk (as measured by variance or standard deviation of return):

- ✓ *Systematic risk*
- ✓ *Unsystematic risk*

Systematic risk is that associated with the market (purchasing power risk, Interest rate risk, liquidity risk, etc.)

Unsystematic risk is unique to an individual asset (business risk, financial risk, Other risks, related to investment into particular asset). Unsystematic risk can be diversified away by holding many different assets in the portfolio; however systematic risk can't be diversified. In CAPM investors are compensated for taking only systematic risk. Though, CAPM only links

Investments via the market as a whole. The essence of the CAPM: the more systematic risk the investor carry, the greater is his / her expected return.

The CAPM being theoretical model is based on some important assumptions:

- All investors look only one-period expectations about the future;
- Investors are price takers and they can't influence the market individually;
- There is risk free rate at which an investors may either lend (invest) or borrow money.
- Investors are risk-averse,
- Taxes and transaction costs are irrelevant.
- Information is freely and instantly available to all investors.

Following these assumptions, the **CAPM predicts what an expected rate of return for the investor should be, given other statistics about the expected rate of return in the market and market risk (systematic risk):**

$$E(r_j) = R_f + \beta_j * (E(r_M) - R_f),$$

Here: $E(r_j)$ - expected return on stock j;

R_f - risk free rate of return;

$E(r_M)$ - expected rate of return on the market

β_j - coefficient Beta, measuring undiversified risk of security j.

Several of the assumptions of CAPM seem unrealistic. Investors really are concerned about taxes and are paying the commissions to the broker when buying or selling their securities. And the investors usually do look ahead more than one period. Large institutional investors managing their portfolios sometimes can influence market by buying or selling big amounts of the securities. All things considered, the assumptions of the CAPM constitute only a modest gap between the theory and reality. But the empirical studies and especially wide use of the CAPM by practitioners show that it is useful instrument for investment analysis and decision making in reality.

Co-efficient Beta (β_j). Each security has its individual systematic - undiversified risk, measured using coefficient Beta. Coefficient Beta (β_j) indicates how the price of security/ return on security depends upon the market forces (note: CAPM uses the statistic measures.

$$\beta_j = \frac{\text{Cov}(r_j, r_M)}{\sigma^2(r_M)}$$

$$\delta^2 (r_M)$$

Arbitrage Pricing Theory (APT)

APT was proposed by Stephen S. Rose and presented in his article, the arbitrage theory of Capital Asset Pricing, published in Journal of Economic Theory in 1976. Still there is a potential for it and it may sometimes displace the CAPM. In the CAPM returns on individual assets are related to returns on the market as a whole. The key point behind APT is the rational statement that the market return is determined by a number of different factors. These factors can be fundamental factors or statistical. If these factors are essential, there to be no arbitrage opportunities there must be restrictions on the investment process. Here *arbitrage* we understand as *the earning of riskless profit by taking advantage of differential pricing for the same assets or security*. Arbitrage is widely applied investment tactic.

APT states, that the expected rate of return of security J is the linear function from the complex economic factors common to all securities and can be estimated using formula:

$$E(r_J) = E(r_J) + \beta_{1J} I_{1J} + \beta_{2J} I_{2J} + \dots + \beta_{nJ} I_{nJ} + e_J,$$

Here:

$E(r_J)$ - expected return on stock J;

$E(r_J)$ - expected rate of return for security J, if the influence of all factors is 0;

I_{iJ} - the change in the rate of return for security J, influenced by economic factor i (i = 1, ..., n);

β_{iJ} - coefficient Beta, showing sensitivity of security's J rate of return upon the factor i (this influence could be both positive or negative);

e_J - error of rounding for the security J (expected value – 0).

It is important to note that the arbitrage in the APT is only approximate relating diversified portfolios, on assumption that the asset unsystematic (specific) risks are negligible compared with the factor risks.

There could presumably be an infinitive number of factors, although the empirical research done by S. Ross together with R. Roll (1984) identified four factors economic variables, to which assets having even the same CAPM Beta, are differently sensitive:

- Inflation;
- Industrial production;
- Risk premiums;
- slope of the term structure in interest rates.

In practice an investor can choose the macroeconomic factors which seem important and related with the expected returns of the particular asset. The examples of possible macroeconomic factors which could be included in using APT model:

- GDP growth;
- An interest rate;
- An exchange rate;
- a default spread on corporate bonds, etc.

Including more factors in APT model seems logical. The institutional investors and analysts closely watch macroeconomic statistics such as the money supply, inflation, interest rates, unemployment, changes in GDP, political events and many others. Reason for this might be their belief that new information about the changes in these macroeconomic indicators will influence future asset price movements. But it is important to point out that not all investors or analysts are concerned with the same set of economic information and they differently assess the importance of various macro economic factors to the assets they have invested already or are going to invest. At the same time the large number of the factors in the APT model would be impractical, because the models seldom are 100 percent accurate and the asset prices are function of both macroeconomic factors and noise. The noise is coming from minor factors, with a little influence to the result – expected rate of return. The APT does not require identification of the market portfolio, but it does require the specification of the relevant macroeconomic factors. Much of the current empirical APT research is focused on identification of these factors and the determination of the factors' Betas. And this problem is still unsolved. Although more than two decades have passed since S. Ross introduced APT model, it has yet to reach the practical application stage. The CAPM and APT are not really essentially different; because they are developed for determine an expected rate of return based on one factor (market portfolio – CAPM) or a number of macroeconomic factors (APT). But both models predict how the return on asset will result from factor sensitivities and this is of great importance to the investor.

Market efficiency theory

The concept of market efficiency was proposed by Eugene Fama in 1965, when his article “Random Walks in Stock Prices” was published in Financial Analyst Journal. *Market efficiency means that the price which investor is paying for financial asset (stock, bond, other security) fully reflects fair or true information about the intrinsic value of this specific asset or fairly*

describes the value of the company – the issuer of this security. The key term in the concept of the market efficiency is the information available for investors trading in the market. It is stated that the market price of stock reflects:

1. *All known information, including:*

- ✓ Past information, e.g., last year's or last quarter's, month's earnings;
- ✓ Current information as well as events that have been announced but are still forthcoming, e.g. shareholders' meeting.

2. *Information that can reasonably be inferred,* for example, if many investors believe that ECB will increase interest rate in the nearest future or the government deficit increases, prices will reflect this belief before the actual event occurs.

Capital market is efficient, if the prices of securities which are traded in the market, react to the changes of situation immediately, fully and credibly reflect all the important information about the security's future income and risk related with generating this income. What is the important information for the investor? From economic point of view the important information is defined as such information which has direct influence to the investor's decisions seeking for his defined financial goals. Example, the essential events in the joint stock company, published in the newspaper, etc. Market efficiency requires the adjustment to new information occurs very quickly as the information becomes known. Obvious, that Internet has made the markets more efficient in the sense of how widely and quickly information is disseminated.

There are 3 forms of market efficiency under efficient market hypothesis:

- Weak form of efficiency;
- Semi- strong form of efficiency;
- Strong form of the efficiency.

Under ***the weak form of efficiency*** stock prices are assumed to reflect any information that may be contained in the past history of the stock prices. So, if the market is characterized by weak form of efficiency, no one investor or any group of investors should be able to earn over the defined period of time abnormal rates of return by using information about historical prices available for them and by using technical analysis. Prices will respond to news, but if this news is random then price changes will also be random.

Under ***the semi-strong form of efficiency*** all publicly available information is presumed to be reflected in stocks' prices. This information includes information in the stock price series as well

as information in the firm's financial reports, the reports of competing firms, announced information relating to the state of the economy and any other publicly available information, relevant to the valuation of the firm. Note that the market with a semi strong form of efficiency encompasses the weak form of the hypothesis because the historical market data are part of the larger set of all publicly available information. If the market is characterized by semi-strong form of efficiency, no one investor or any group of investors should be able to earn over the defined period of time abnormal rates of return by using information about historical prices and publicly available fundamental information(such as financial statements) and fundamental analysis.

The strong form of efficiency which asserts that stock prices fully reflect all information, including private or inside information, as well as that which is publicly available. This form takes the notion of market efficiency to the ultimate extreme. Under this form of market efficiency securities' prices quickly adjust to reflect both the inside and public information. If the market is characterized by strong form of efficiency, no one investor or any group of investors should be able to earn over the defined period of time abnormal rates of return by using all information available for them. The validity of the market efficiency hypothesis whichever form is of great importance to the investors because it determines whether anyone can outperform the market, or whether the successful investing is all about luck. Efficient market hypothesis does not require to behave rationally, only that in response to information there will be a sufficiently large random reaction that an excess profit cannot be made. The concept of the market efficiency now is criticized by some market analysts and participants by stating that no one market can be fully efficient as some irrational behavior of investors in the market occurs which is more based on their emotions and other psychological factors than on the information available. But, at the same time, it can be shown that the efficient market can exist, if in the real markets

Following events occur:

- ✓ A large number of rational, profit maximizing investors exist who are actively and continuously analyzing valuing and trading securities;
- ✓ Information is widely available to market participants at the same time and without or very small cost;
- ✓ Information is generated in a random walk manner and can be treated as independent;
- ✓ Investors react to the new information quickly and fully, though causing market prices to adjust accordingly.